Flocking Behaviour and Information Flow of the Topological Vicsek Model

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Abstract

In this work we investigate the link between the phase transition from ordered to disordered states in the Vicsek model of self-propelled particles and the information theory metrics – mutual information (MI) and transfer entropy (TE) – surrounding the flock. Prior work has investigated this link in one variant of the Vicsek Model, known as the Metric method, and has found that both MI and TE peak at the phase transition. This work aims to explore whether this link exists for another variant known as the Topological method. We also discuss the computational complexities and solutions to collecting this data.

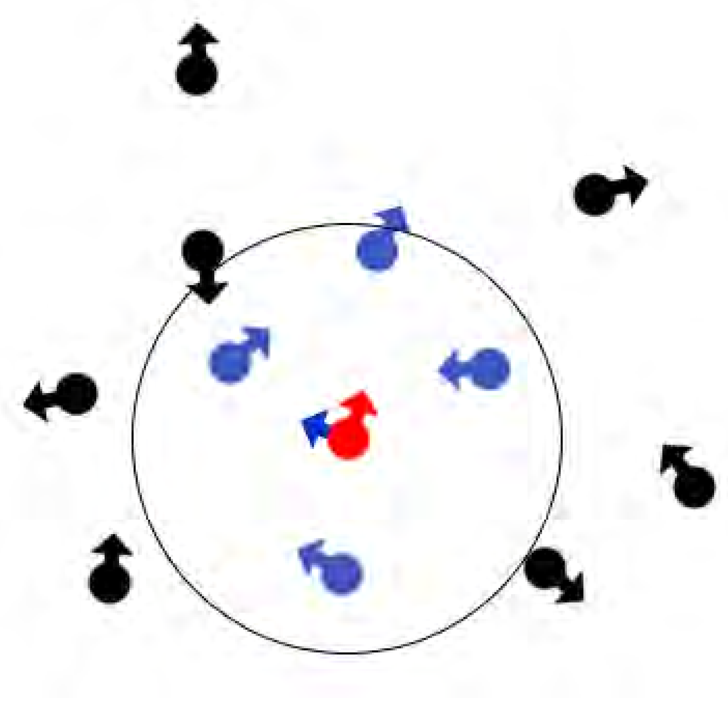

Fig 1a: The metric variant of the Vicsek Model. Red particle assumes average direction of itself and all particles within radius r (blue particles)

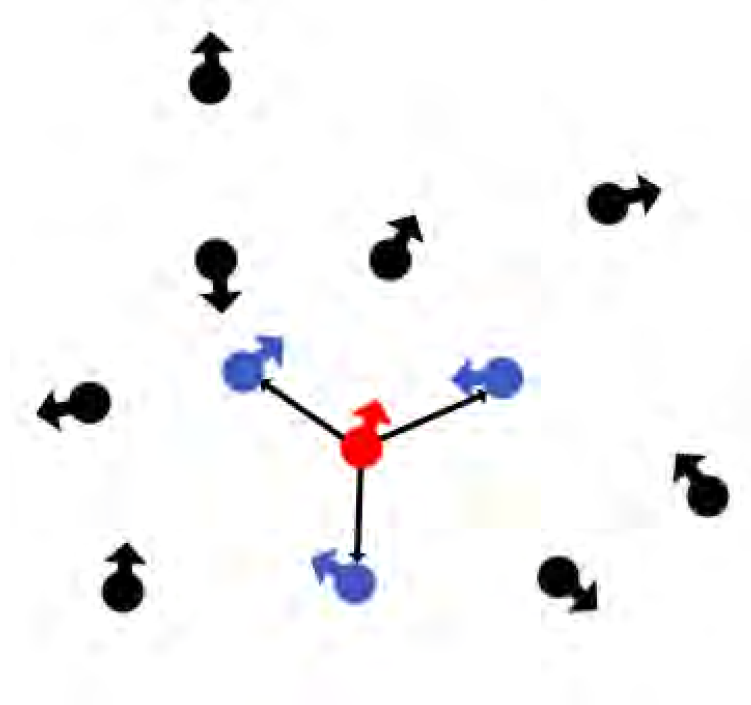

Fig 1b: The topological variant of the Vicsek Model. Red particle assumes average direction of itself and n=3 closest particles (blue particles)

Motivation and Problem Statement

In the Vicsek model, at each time step, each particle assumes the average heading of its neighbours (including itself) plus some random perturbation. Fig 1 shows the two variants of the Vicsek model considered and the difference in how particles determine neighbours.

Fig 2 shows the effect the extreme values of random perturbation can have on the flock structure. At some intermediate value, the flock will transition between the two extreme states, at a point known as the phase transition, or critical state.

Prior work has found that Mutual Information (Fig 3) and Transfer Entropy (Fig 4) of a metric Vicsek flock peaks at the phase transition (Fig 5). This work investigates if the same behaviour is found in the topological variant as well.

Fig 2a: No random perturbation added to particle heading. Particle headings eventually converge to the same direction, creating an ordered flock.

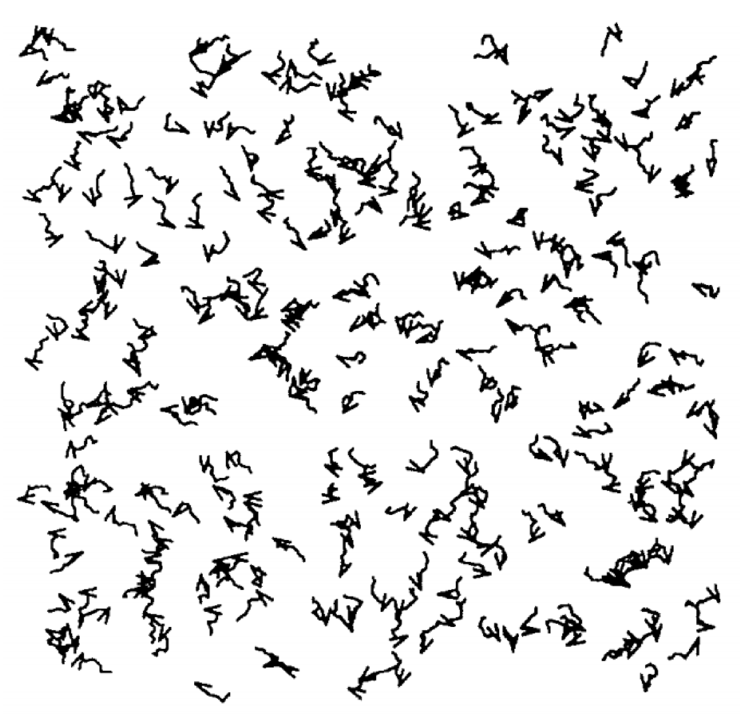

Fig 2b: Maximum random perturbation added to particle heading. Particle headings are dominated completely by noise, and travel in random directions at each time step, signalling a disordered flock.

Experimental Results and Conclusions

Fig 6 shows preliminary results of topological variant with n=5. Results from fewer noise values and fewer repetitions per noise value. Further tests with increased ranges required. Additionally, further tests with various n values also required.

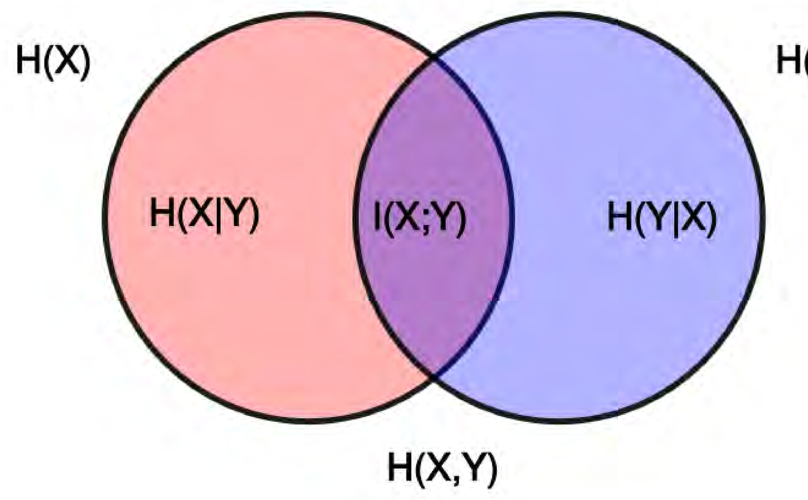

Fig 3: Shows information, or marginal entropies, H(X) and H(Y), of particles X and Y. Entire region shows joint entropy, H(X,Y). Purple region labeled I(X;Y) shows the overlap or sharing of information, giving the Mutual Information. This is given by the equation: I(X;Y) = H(X) + H(Y) – H(X,Y)

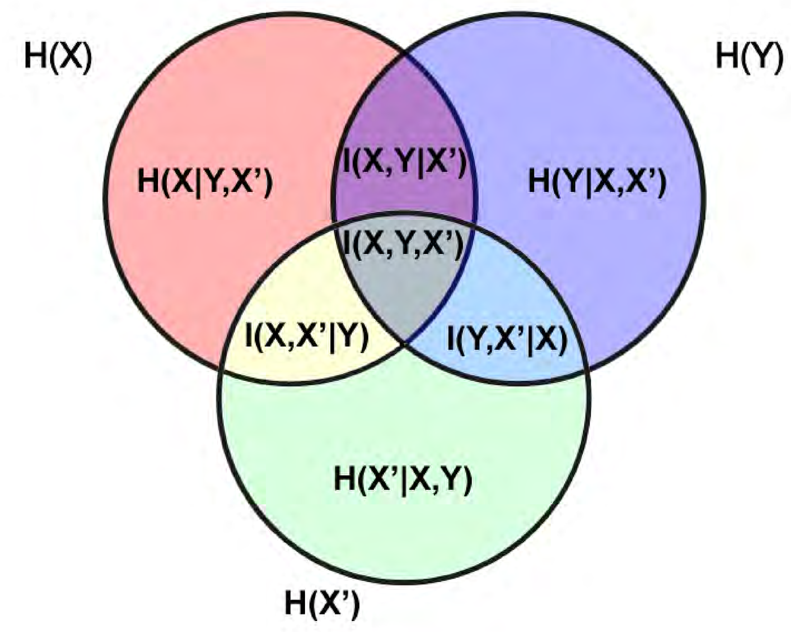

Fig 4: Transfer Entropy measures the flow of information from past value of Y to current value of X (X'). I(X':Y) gives the grey and teal regions, however, this also includes the flow of information from X's past, so needs to be conditioned out, to give just the teal region, i.e. I(X':Y|X)

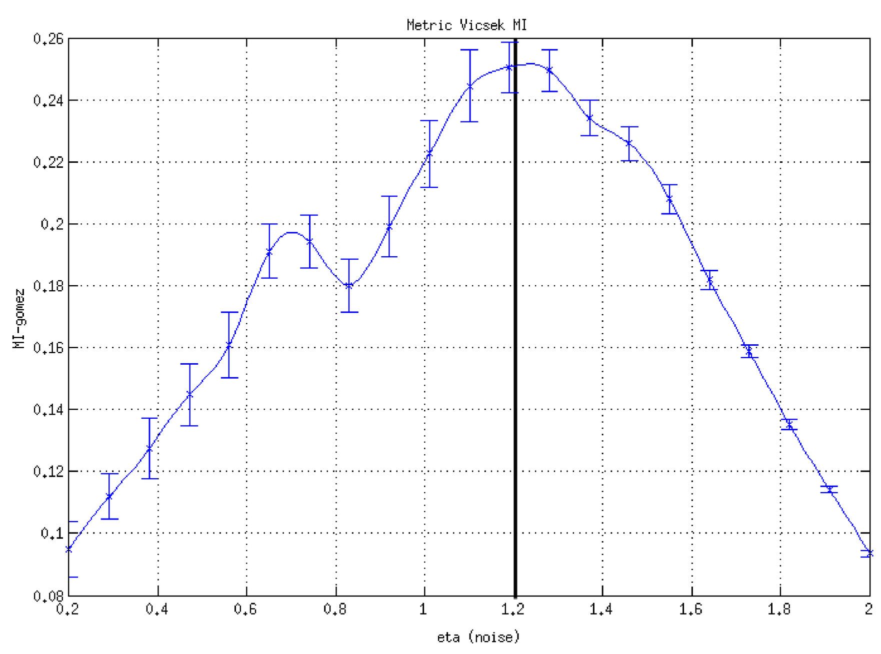

Fig 5: MI peaking at phase transition (indicated by line) in the metric model

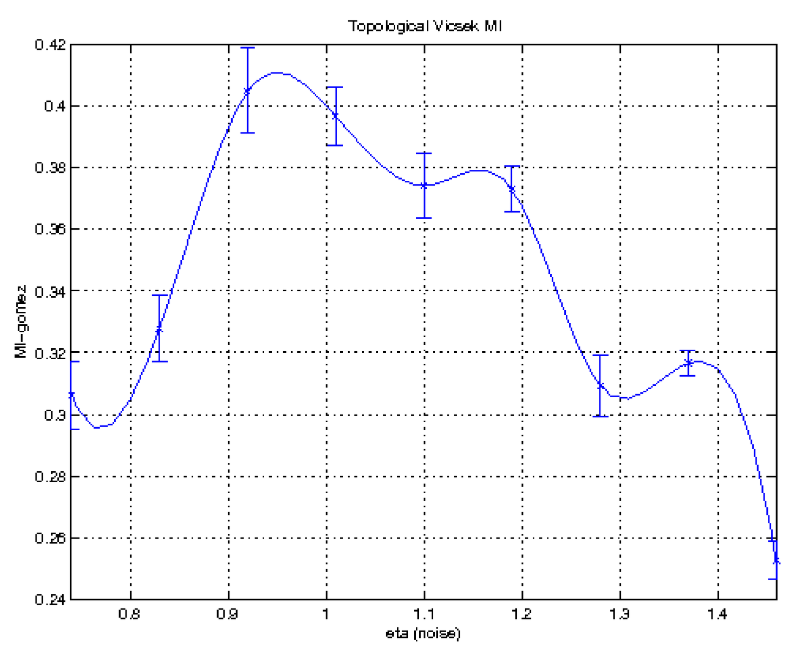

Fig 6: Preliminary results for MI of topological approach with 5 neighbours. MI seems to peak around phase transition. More simulations required to reduce error in results as well as investigate effect of n on the MI and TE.

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