



# Fractions: Theory and examples

When we talk about a fraction, we are saying that it is a part of a whole. An understanding of fractions and how to add, subtract, multiply and divide is a foundation for work in primary mathematics. Explanations of some of the terms that have been used in this document can be found in the glossary on our website. Part 2 of this document contains exercises to practise.

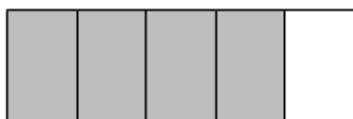
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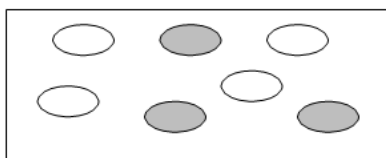
## 1. Definition of a fraction

A fraction is a part of a whole. The fraction  $\frac{4}{5}$  can be demonstrated by shading 4 portions of a whole that has been split into 5 parts.

$$\frac{4}{5}$$



$$\frac{3}{7}$$



**Note:** Numerator is the top number in the fraction. Denominator is the bottom number in the fraction.

**Note:** Depending on your calculator your fraction button could look like  $a \frac{b}{c}$  or  $\frac{\blacksquare}{\blacksquare}$

The fraction bar such as in  $\frac{2}{5}$  means the same as  $2 \div 5$ .

## 2. Types of fractions

There are three types of fractions:

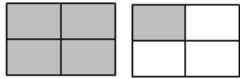
- A **proper fraction** has a smaller numerator than the denominator so represents a portion less than one whole: for instance,  $\frac{1}{2}$
- An **improper fraction** has a numerator bigger than the denominator so represents a portion more than one whole: for instance,  $\frac{11}{3}$
- A **mixed numeral** has a whole number as well as a fraction part: for instance,  $2\frac{1}{3}$

## 3. Converting between different types of fractions

Converting a mixed number to an improper fraction.

Examples:

a)  $1\frac{1}{4}$



As the denominator here is 4 then one whole represents  $\frac{4}{4}$

$$\text{Therefore } 1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

**Note:** to calculate the numerator.

When converting from a mixed to an improper fraction:

1. Multiply the whole number of the mixed fraction by the denominator
2. Add on the numerator of the fraction part

b)  $2\frac{1}{3}$



As the denominator here is 3 then one whole represents  $\frac{3}{3}$

$$\text{Therefore } 2\frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$$

c)  $2\frac{1}{6} = \frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{13}{6}$

d)  $3\frac{4}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5} = \frac{19}{5}$

## Converting an improper fraction to a mixed numeral

Examples:

a)  $\frac{14}{4}$

One whole represents  $\frac{4}{4}$  so there are  $\frac{4}{4}$  &  $\frac{4}{4}$  &  $\frac{2}{4}$  left over

or  $\begin{array}{r} 3r2 \\ 4 \overline{)14} \end{array}$  so  $\frac{14}{4} = 3\frac{2}{4}$

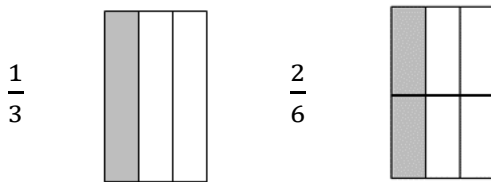
b)  $\frac{8}{5}$

Divide 8 by 5  $\begin{array}{r} 1r3 \\ 5 \overline{)8} \end{array}$

The result is 1 and 3 left over so  $\frac{8}{5} = 1\frac{3}{5}$

## 4. Equivalent fractions

Two fractions that are equivalent represent exactly the same portion of the whole.



This process either breaks the whole up into smaller portions or expresses the fraction in the simplest or most basic form.

If you multiply both the number and denominator of a fraction by a number then you chop the basic fraction up into smaller portions.

Examples:

a)  $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$  (as shown above)

b)  $\frac{1 \times 10}{3 \times 10} = \frac{10}{30}$

The value of the fraction is unchanged because you are multiplying by the equivalent of 1.

$\left(\frac{2}{2} \text{ or } \frac{10}{10}\right)$

Equivalent fractions are often formed when we are adding and subtracting fractions.

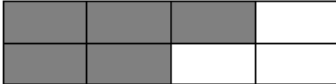
**Note:** When working with fractions remembering that what you do to the top you must do to the bottom!

## 5. Adding and subtracting fractions

Adding and subtracting fractions is a process of collecting together like fractions. If the denominators are the same (like fractions) then just add or subtract the numerators.

Examples:

a)  $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

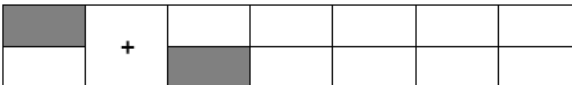


b)  $\frac{7}{11} - \frac{3}{11} = \frac{4}{11}$



When the denominators are different you must form equivalent fractions to make the denominators the same.

c)  $\frac{1}{2} + \frac{1}{10}$

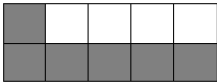


First find the common multiples of the denominators. This will let us find the lowest common denominator (LCD)

2: (2, 4, 6, 8, **10**, .... (like times tables)

10: (**10**, 20, 30, .... (10 is the LCD – convert both fractions to 10ths)

We can look at the steps that need to be taken:

$$\begin{aligned} \frac{1}{2} + \frac{1}{10} & \quad \text{LCD} = 10, \text{ both denominators become } 10 \\ & = \frac{1 \times 5}{2 \times 5} + \frac{1}{10} \quad \text{To make } \frac{1}{2} \text{ into } \frac{?}{10} \text{ multiply top and bottom by } 5 \\ & = \frac{5}{10} + \frac{1}{10} \quad \text{Now the fractions are like so add the numerators.} \\ & = \frac{6}{10} \end{aligned}$$




d)  $\frac{3}{4} - \frac{1}{3}$  Multiples 4: 4, 8, **12**, 16, ....

3: 3, 6, 9, **12**, .....

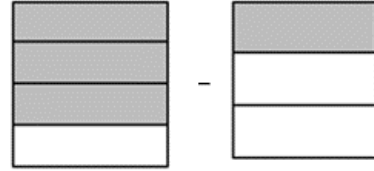
12 is the LCD – convert both fractions to 12ths

To make  $\frac{3}{4}$  into  $\frac{?}{12}$  multiply top and bottom by 3.

To make  $\frac{1}{3}$  into  $\frac{?}{12}$  multiply top and bottom by 4.

$$\frac{3}{4} - \frac{1}{3}$$

Lowest Common Denominator = 12

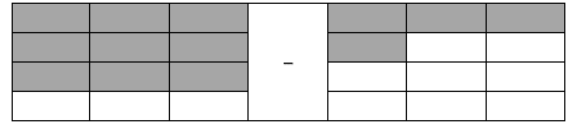


$$= \frac{3 \times 3}{4 \times 3} - \frac{1 \times 4}{3 \times 4}$$

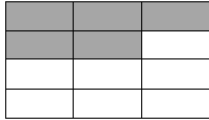
Form equivalent fractions to make each denominator = 12

$$= \frac{9}{12} - \frac{4}{12}$$

Subtract the numerators of the like fractions.



$$= \frac{5}{12}$$



e)  $\frac{8}{9} - \frac{5}{6}$  LCD = 18

$$= \frac{8 \times 2}{9 \times 2} - \frac{5 \times 3}{6 \times 3} = \frac{16}{18} - \frac{15}{18} = \frac{1}{18}$$

## 6. Simplifying fractions

When working out a fractions question, the answer should be presented as a proper fraction expressed in its simplest form or as a mixed numeral expressed in its simplest form.

To simplify a fraction, divide the numerator and denominator by a common factor.

**Note:** a common factor is a number that divides evenly into both the numerator and denominator. Again, this is equivalent to dividing by 1 so will not change the value of the fraction.

**Examples:**

a) For part C in section 5, the answer of  $\frac{6}{10}$  can be simplified as follows:

$\frac{6}{10}$  2 is a factor of 6 and 10 so we divide top and bottom by 2

$$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

b)  $\frac{20}{25}$  5 is a factor of 20 and 25 so we divide top and bottom by 5

$$\frac{20 \div 5}{25 \div 5} = \frac{4}{5}$$

c)  $\frac{80}{140}$  10 is a factor of 80 and 140 so we divide top and bottom by 10

$$\frac{80 \div 10}{140 \div 10} = \frac{8}{14} \text{ or } \frac{8\cancel{0}}{14\cancel{0}} = \frac{8}{14}$$

Then we look at the factors again. 2 is a factor of 8 and 14.

$$\frac{8 \div 2}{14 \div 2} = \frac{4}{7}$$

d)  $\frac{36}{16}$        $\frac{36 \div 4}{16 \div 4} = \frac{9}{4}$

As this answer is an improper fraction, we can convert it to a mixed numeral.

$$\frac{9}{4} = 2\frac{1}{4}$$

## 7. Multiplying fractions

Multiplying fractions is a way of chopping the fraction up.

**Examples:**

a)  $\frac{1}{2} \times \frac{1}{2}$  means find  $\frac{1}{2}$  of  $\frac{1}{2}$



$$= \frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

**Note:** The rule when multiplying fractions is to multiply the numerators and multiply the denominators. Then simplify the answer if possible.

b)  $\frac{1}{3} \times \frac{3}{10}$  means find  $\frac{1}{3}$  of  $\frac{3}{10}$

$= \frac{3}{10}$

$$= \frac{1}{3} \text{ of } \frac{3}{10} = \frac{1}{3} \times \frac{3}{10} = \frac{1 \times 3}{3 \times 10} = \frac{3 (\div 3)}{30 (\div 3)} = \frac{1}{10}$$

$$\text{or } \frac{1}{3} \times \frac{3}{10} = \frac{1 \times 3 (\div 3)}{3 (\div 3) \times 10} = \frac{1 \times 1}{1 \times 10} = \frac{1}{10}$$

**Note:** an alternative method here is to cancel or simplify before multiplying out.

$$\text{c) } 4 \times \frac{5}{7}$$

$$4 \times \frac{5}{7} = \frac{4}{1} \times \frac{5}{7} = \frac{4 \times 5}{1 \times 7} = \frac{20}{7} = 2\frac{6}{7}$$

**Note:** any whole number can be written as a fraction by using 1 as the denominator.

$$\text{d) } \frac{3}{8} \times 20 = \frac{3}{8} \times \frac{20}{1} = \frac{3 \times 20}{8 \times 1} = \frac{60}{8} \begin{matrix} (\div 2) \\ (\div 2) \end{matrix} = \frac{30}{4} \begin{matrix} (\div 2) \\ (\div 2) \end{matrix} = \frac{15}{2} = 7\frac{1}{2}$$

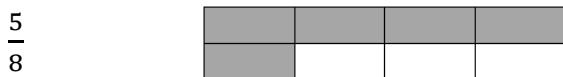
$$\text{or } \frac{3}{8} \times \frac{20}{1} = \frac{3 \times 20 (\div 4)}{8 (\div 4) \times 1} = \frac{3 \times 5}{2 \times 1} = \frac{15}{2} = 7\frac{1}{2}$$

## 8. Dividing fractions

When dividing fractions, you are asking how many shares you have.

**Examples:**

a)  $\frac{5}{8} \div \frac{1}{4}$  is asking how many one quarter shares there are in  $\frac{5}{8}$



**Asking:** How many  $\frac{1}{4}$  s in  $\frac{5}{8}$ ?

Each  $\frac{2}{8}$  ths represents a one quarter share, so  $\frac{5}{8}$  ths represents just over 2 shares.

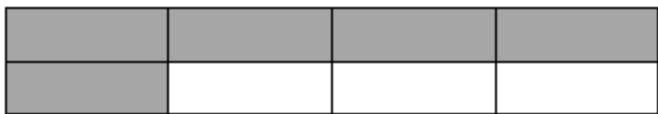
$$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{1} = \frac{5 \times 4}{8 \times 1} = \frac{20}{8} \begin{matrix} (\div 4) \\ (\div 4) \end{matrix} = \frac{5}{2} = 2\frac{1}{2}$$

$\div \frac{1}{4}$  is the same as  $\times 4$

**Rule:**

1. Invert the second fraction.
2. Multiply the two fractions.
3. Then simplify.

b)  $\frac{1}{4} \div \frac{5}{8}$  is asking how many five eighths shares there are in  $\frac{1}{4}$



$\frac{5}{8}$  is a bigger fraction than  $\frac{1}{4}$  so the answer will be less than one.

$$\frac{1}{4} \div \frac{5}{8} = \frac{1}{4} \times \frac{8}{5} = \frac{1 \times 8}{4 \times 5} = \frac{8 (\div 4)}{20 (\div 4)} = \frac{2}{5}$$

**Note:** To multiply and divide mixed numerals you must first convert each fraction to its improper form or use a calculator.

$$\text{c) } \frac{3}{11} \div 4 = \frac{3}{11} \div \frac{4}{1} = \frac{3}{11} \times \frac{1}{4} = \frac{3 \times 1}{11 \times 4} = \frac{3}{44}$$

$$\text{d) } 5 \div \frac{6}{8} = \frac{5}{1} \div \frac{6}{8} = \frac{5}{1} \times \frac{8}{6} = \frac{5 \times 8 (\div 2)}{1 \times 6 (\div 2)} = \frac{5 \times 4}{1 \times 3} = \frac{20}{3} = 6 \frac{2}{3}$$

## 9. For more information

Visit our [intro to maths](#) page on the Charles Sturt Student Portal where you can access more mathematics and numeracy resources, find and register for our Enhancing Numeracy workshops or make a 1:1 appointment with our numeracy advisers.