Classification over Grassmann Manifold

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Abstract

Smooth Riemannian manifolds, such as Grassmann manifolds(coordinate systems of subspaces of a space) have widely been used in modern computer vision applications, e.g., classification and clustering. The existing classification methods such as Support Vector Machine(SVM) and Distance Weighted Discrimination(DWD), and many other conventional methods unfortunately do not handling data lying on manifolds.

In this paper, we present an effective and simple Naïve Bayes classification approach for constructing classifier models to problem instances[1]. For modeling manifold valued data there exists only few exponential families of distributions, among which the Bingham antipodal symmetric distribution is most tractable for Grassmann manifolds.

Although, the exponential families of distributions are seemingly very pleasant, however the normalizing constant associated with them is practical difficulty for the tasks of even simple maximum likelihood estimations. For our purpose to overcome this difficulty we use the first few important terms of the asymptotic series expansion of Bessel function for special case of small values of concentrated parameters in our inference models.

Grassmann Manifolds

The set of p-dimensional subspaces of R^n is called the Grasssmann manifold denoted by $G_{n,p}$. Since a subspace is specified by the orthogonal projection onto it, $G_{n,p}$ can be described in terms of nxn matrices X:

$$G_{n,p} = \{X : X = X^T = X^T X, Rank(X) = n\}$$

Parametric Model over Grassmann Manifolds

The matrix Bingham distributions on $G_{n,p}$ are generalizations of the Bingham distributions on real projective space RP^{n-1}. For random samples of matrices $X_1,\ldots, X_n$, sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ the density function [4] is

$$F(X \mid B) = \frac{1}{B_p(D)} exp(trace[(P_S X)^T B (P_S X)])$$

where $B_p(D) = \frac{I_{p/2}(D)}{I_{\frac{p}{2}-1}(D)}$ is modified Bessel function of kind first and order p.

S is a subspace of R^n of dimension p, $P_S = UU^T$ orthogonal projection matrix onto S, and $B = UDU^T$ is spectral decomposition.

For small values of sorted monotonically decreasing singular values $D = diag(d_1, \ldots, d_n)$, we considered the expansion of $B_p(D)$ for our inference model as [2]:

$$B_p(D) = \frac{1}{p}(D) - \frac{1}{p^2(p+2)} D^3 + O(D^5)$$

The expression for large D is:

$$B_p(D) = 1 - \frac{p-1}{2}\frac{1}{D} + \frac{(p-1)(p-3)}{8}\frac{1}{D^2} + O(\frac{1}{D^3})$$

Experimental Process and Results

Test, of 3-classes classification problem chosen from 36 classes of the data set(177x14x3600).
Classes are denoted by Group(1,2,3), using our Algorithm on the data set, we used 20 points from each group, and the rest of the data for inferences. Accordingly we calculated the error rate for our model on Grassmann manifold as 1.8%.

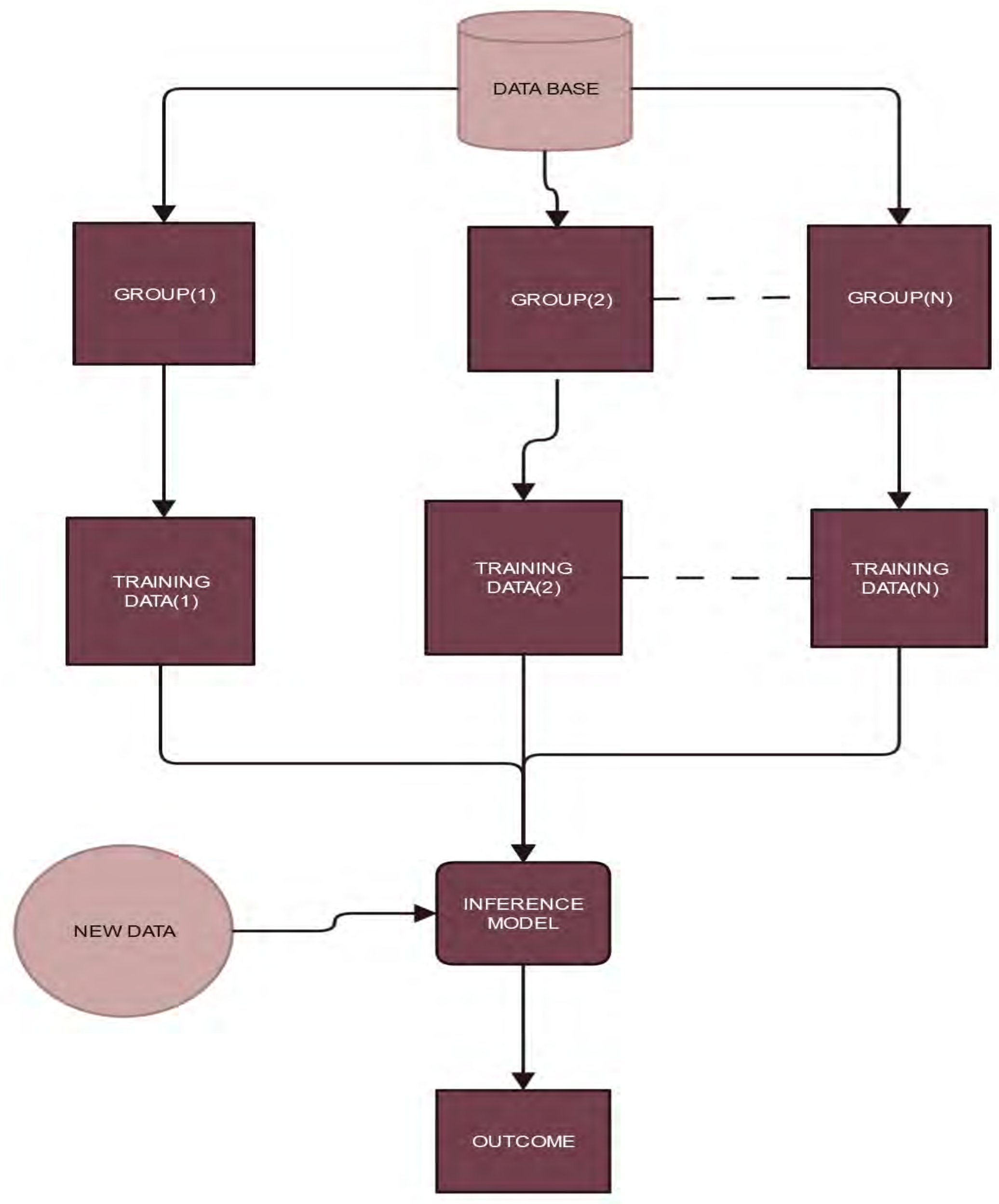

Fig 1: graphical outlines which express our step by step algorithmic computations. In our experiment we have 3600 matrices of size 177x4 as points on Grassmann manifold. We randomly choose 36 classes of the data set, and consider 3-classes classification problem as an example.

Fig 1: DynTex++ Samples (Each row is from the same video sequence). This database [3] is derived from a total of 345 video sequences in different scenarios and are labeled as 36 classes. Each class has 100 subsequences with fixed size of 50x50x50(50 gray frames). We consider Local Binary Patterns from Three Orthogonal Plans(LBP-TOP) model and used it for generating points on Grassmann manifold.

References:

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[3] B. Wang, Y. Hu, J. Gao, Y. Sun and B. Yin. Kernelized low rank representation on Grassmann manifolds. IEEE Trans on Pattern Anal and Machine Intel., Submitted(X), 2015.
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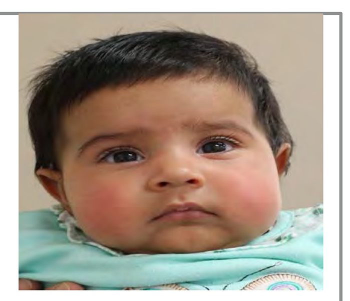

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